Understanding Uncertainty in Prediction Methods

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ABSTRACT

In today's environment of customers demanding more efficient and affordable cost of ownership, it is imperative that we improve and standardize our technical assessment approach. Historically, solid rocket motor (SRM) analysts have evaluated design integrity based on a safety factor (SF) or margin of safety (MS) design criteria during the design and production phase of the life cycle. This approach obtains capability or failure limits through testing to failure and applies degradation factors (i.e., 3σ values used), but the induced loads or requirements are typically calculated using numerical methods such as finite element using characterization data obtained from laboratory testing. We often apply conservative factors (i.e., 3σ values) to the input properties and use conservative material models when determining whether or not our design meets customer SF or MS requirements. If the SF or MS calculation, with this conservatism, meets the requirement we typically document that and move on to the next item.

In reality, this approach does not enable us to rigorously calculate uncertainties. It is a "good enough" approach that has been universally accepted and proven successful for production purposes, but is not adequate for assessing effects of aging. The desire to more accurately predict the useful life cycle drives us to a more rigorous and accurate assessment approach that identifies and deals with uncertainty associated with all aspects of the prediction methodology. For example, using improved analytical approaches that quantify how close the induced loads get to failure allows us to quantify uncertainty and achieve improved service life predictions. The "good enough" approach often used for design and production is not adequate for service life prediction since it does not rigorously address uncertainty and its sources.

1. INTRODUCTION

Collection of accurate health state information about a product is crucial to the success of our products.

Systems engineering plays a vital role in this technical process. The systems engineering processes "*are used to define the requirements for a system, to transform the requirements into an effective product, to permit consistent reproduction of the product where necessary, to use the product to provide the required services, to sustain the provision of those services, and to dispose of the product when it is retired from service*." [1]

These processes define the activities to optimize the designs and reduce the risks associated with technical decisions during the development, manufacturing, fielding, operation, and removal from service of the products. These processes also define, verify, validate, and control the "*the timeliness and availability, the cost effectiveness, and the functionality, reliability, maintainability, producibility, usability, and other qualities*" [1] of the system. An extremely important part of this process is to accurately ascertain the ability of the product to meet the required performance throughout its life. An important part of this process is to capture information about the product through testing, component and system model analysis, and collection of information from the systems performance, which contains key parameters as defined by the performance models of the system.

If information is collected incorrectly or inaccurately or the method of collection does not provide the information to accurately model the behavior, then the performance of the model will have increased uncertainty (may not be understood or cannot be quantified) or may not represent the expected behavior at all. In today's environment of customers demanding more efficient and affordable designs while maintaining or increasing reliability, with lower mass and lower overall cost of ownership, it is imperative that we improve and standardize our technical assessment approach.

Historically, solid rocket motor (SRM) analysts have evaluated design integrity based on a safety factor or margin of safety design criteria. This approach obtains capability or failure limits through testing to failure and applies degradation factors (i.e., lower 2 or 3σ values used).The induced loads are typically calculated using numerical methods such as finite element using characterization data obtained from laboratory testing. We often apply conservative factors (i.e., 3σ values) to the input properties and conservative material models when determining whether or not our design meets customer safety factor or margin of safety requirements. If the safety factor or margin of safety calculation, with this conservatism, meets the requirement, we typically document that and move on to the next item. In reality, this approach does not enable us to calculate uncertainties. It is a "good enough" approach that has been universally accepted and proven successful for design.

The industries' push to provide new capabilities that insure success with common designs that provide enhanced performance, commonality, affordability, and service life drive us to be more rigorous and accurate in the assessment of our designs. For example, using improved analytical approaches that quantify how close the induced loads get to failure allows us to quantify uncertainty and achieve more affordable and optimized designs. This approach fosters modularity and enables use of the designed products to be safely repurposed. The "good enough" approach often leads to heavy and more costly solutions whereas a more rigorous approach enables confidence in reducing component weight and cost without increased risk.

Transitioning to a physics of failure mentality that allows for optimal designs to be created with known errors (uncertainty) in their output and hence a better understanding of the limits of the design is where we are headed. This change in the culture must occur in order for our products to be competitive and still perform with the high expectations of the community today and in the future. This approach does not sacrifice safety or performance margin, rather it quantifies the relationship of product performance with an understanding of the true factor or margin of safety and uncertainties.

In order for this transition to occur, an understanding of how our models are used to accurately predict the performance of the product is crucial. This requires that the data collected be understood, the model be anchored and used within its bounds of validation, and a sensitivity analysis accomplished to understand the parameter combinations (material, environment, loads) which may adversely affect performance. This transition is not easy and requires validation testing to adequately anchor the approaches. Data collection must also be accomplished with testing that represents the materials' use in its intended application and all data collected must be quantified as to its accuracy and variability including describing how the test is accomplished and the data is collected.

The fundamental challenge of accurately predicting a SRM's performance throughout the life-cycle includes accurate prediction of material and subcomponent performance. SRM performance at the system level is most often not affected by age-related changes in materials or subcomponents until those materials or subcomponents fail leading to a system performance issue. Age related changes are often exhibited at the system level as a step change in performance and not a gradual degradation in system performance until failure is ultimately experienced.

This aspect of predicting SRM performance does not allow us to address uncertainties unless those items that are degrading are addressed and their contribution to the system-level uncertainties are quantified. The desire to design, qualify, operate, and sustain reliable SRMs with a reduced amount of full-scale destructive testing and targeted relevant subscale testing is driving the need for better understanding of the limits of the materials and for quantifying uncertainties in the data collection and analysis processes.

2. BACKGROUND

There are two distinct uses for safety factor (SF): One is a ratio of **absolute strength** (structural capacity) to **actual applied load** which is similar to what some refer to a nominal probability of failure (P_f). It is important to note that the SF calculation is a single number. This method, of using absolute values, is not however typically used for design. The other use of SF is a constant value imposed by law, standard, specification, contract or custom to which a structure must conform or exceed. SRMs are typically designed to the second type use and are purposefully built much stronger than needed for normal usage to allow for degradation. This is the method Orbital ATK's customers have typically invoked for designing an SRM.

There are several ways to compare the safety factor for structures. All the different calculations fundamentally measure the same thing, which is to determine how much extra load beyond what is intended for the design will a structure actually take (or be required to withstand at maximum load). Equation 1 is the industry standard used for SF calculation, but for conservatism, degradation factors are often used by the aerospace industry due to the enhanced focus on reduced risk for high value assets. The difference between the methods is the way in which the values are calculated and compared. Safety factor values can be thought of as a standardized way for comparing strength and reliability between systems.

$$\text{Safety Factor} = \frac{\text{Material Strength}}{\text{Design Load}} \qquad \text{Eq. 1}$$

Historically, there has been a push towards conservatism in SRM design in the calculation of safety factors, i.e., in the absence of highly accurate data, using the worst case possible to make sure the system is adequate (to err on the side of caution).

This approach leads to a conservatism for all components, but does not necessarily apply the same conservatism to all components of the SRM. For example, material properties and their inherent variability from standardized tests performed in the laboratory, which when used to predict full-scale motor performance often require a correction factor to be applied to the data used in the analysis. Another example is if an analyst erroneously selects an input parameter for use in the analysis that is not appropriate for the analysis or does not adequately address the uncertainty in the parameter resulting in an inaccurate prediction. This can also lead to variable analysis predictions from multiple analysts. The problem here is also expanded when analysts select a property that is not representative of the conditions the material is under in the analysis. Property selection can be difficult since the actual motor conditions can be extreme and are often not well understood.

As SRMs age, owners need to know how safe they are with degraded or aged properties and the testing and analysis shifts from a design consideration to a P_f approach where accuracy becomes paramount.

Margin of safety (MS) is typically what aerospace uses to describe the ratio of the strength of the structure to the requirements because most U. S. government agencies invoke MS requirements where they specify the minimum SF allowable. Equation 2 shows the simplest form of the Ms. In theory, if the MS is 0 then the part will not take any additional load before it fails and if MS is negative, the part should fail before reaching its design load. If the MS is 1, the part can withstand two times its design load.

$$Margin\ of\ Safety\ = \frac{Failure\ Load}{Design\ Load} - 1 \qquad \text{Eq. 2}$$

It should be obvious that MS, similar to SF, can be used for conservative design or for nominal P_f types of calculations depending on the approach used to determine the failure and design loads. Again, it is important to note that the MS calculation results in a single number.

The aerospace community typically uses some variation of Equation 3 for MS calculations where the design load is multiplied by the design SF. Propellant grain structural analysts typically use degraded material strength properties and worst case design loads when calculating MS adding further to the conservatism.

$$Margin\ of\ Safety\ = \frac{Material\ Strength}{Design\ Load * Design\ SF} - 1 \qquad \text{Eq. 3}$$

SRM design following the SF and MS approach has led to very successful motor designs over the years. However, there is a strong push in the SRM community to become more affordable. Customers and Orbital ATK management are starting to ask designers and analysts to quantify the uncertainty in their methods. Most of the SRM designers and analysts are confounded by this question and with good reason. Our industry has historically dealt with the “design SF” or “design MS” approaches, which have not been forced to deal with quantifiable uncertainties as long as the analyst has been able to show his/her approach is similar to those that have been accepted in the past. In order to rigorously deal with uncertainty, one needs to better understand the sources of uncertainty and the sensitivity effects on performance of the materials or components. This process forces the analysis to be more accurate in its representation of the material or components to their actual operation and exposure to environments. This quantification allows for the analysis to bound errors that exist and provide for more confidence in their solutions. This then enables a consistent application of conservative factors to be applied. This process allows for some optimization of design over the current approach, but a transition to P_f calculations along with validation testing that allows calculating the percent error or uncertainty in the methods is required to successfully address the goals of the customers.

3. SYSTEMS ENGINEERING APPROACH

Systems engineering is used to allocate and define requirements of the system, and allocate the appropriate requirements to subsystem, component, and materials. In this process, it is important to capture the performance requirements of each component and material and its relationship to the system operation. This process is part of the transformation of the levied requirements into an effective product. During development, it is important to capture each of the interfaces for each element of the system.

These interfaces include: structural, chemical, electrical, thermal, and performance interfaces. Each of these interfaces serves a purpose for success of the system, whether it is to define environmental loads applied to the elements, chemical interfaces between elements, power or communication transmission between the elements, or performance of the element. Each of these interfaces identifies some interaction with the element and its neighbor. In order for a system product to be successful, all of the interfaces need to be managed to insure that each is in compliance with the interaction requirements of the connected components.

During deconstruction of the system, including allocating each requirement and defining all interfaces to each component, subcomponent, and material of the system, an understanding can be obtained of what parameters are critical to the performance of each subcomponent and its effect on its parent component. This allows for the systems' critical performance-based parameters to be ascertained. Note it is an important part of systems engineering development to capture performance-based parameters at the system and component levels along with any adjustments to functional requirements required by subcomponent capabilities. Assessing the compliance to these requirements requires capture of information relative to the performance of the item.

Assessing the performance of the system is accomplished by physical models of all components that contribute to the system's success while exposed to operational conditions. These material, component, and system models use material properties to assess the performance of the system. Accurately capturing information about the system is necessary to insure the predicted performance can meet customer requirements and perform for the life of the system.

Once this deconstruction of the system is complete, including allocating each requirement and defining all interfaces to each component and subcomponent of the system, an understanding can be obtained of what parameters are critical to the performance of each subcomponent and its effect on its parent component. This understanding comes from modeling of the components' behavior under the expected operational conditions. This allows for the systems' critical performance-based parameters to be ascertained and a plan to capture the requisite data to be obtained by a Prognostic Health Management (PHM) system.

Note it is an important part of systems engineering development to capture performance-based parameters at the system and component levels along with any adjustments to functional requirements required by subcomponent capabilities. Models that are used for this process must be representative of the component's behavior and must define the relationship of each parameter that affects the component's ability to meet performance requirements. Capturing this information in the performance requirements for the components is necessary for system success and must include each critical parameter that is required to assess performance of the system today and into the future. This process places an element of component and measurement realism into allocated system functional requirements. The verification/validation process is also a necessary and time-consuming process that must be done on the components and their models first and then progressing up through the full system. This process is represented by a systems engineering V diagram [Figure 3-1].

Capturing a system's performance as a function of each component of the system is required to insure the system's requirements and the performance expectations of the customers are met. An important part of capturing the performance of each component involves understanding the risk to the system performance associated with each component's ability to meet its requirements. This process requires an understanding of each model's performance prediction and the uncertainty or error associated with the model relative to the actual performance of the product during its mission. [2] The uncertainty of each component's model to meet the allocated performance criteria can be assessed using the error role-up process. [3] This includes capturing the associated errors in the material property data, the assumptions used to generate the model, and the model's performance relative to a representative test case used to anchor the models.

4. PROBABILITY OF FAILURE APPROACH

We learned from Section 2 that our industry adopted the SF and MS design approaches. We know from the successful performance history of SRMs that these methods have worked well from a design standpoint. It was pointed out previously that both SF and MS equation calculations provide a single number. The variability, or uncertainty, of the solution is essentially buried in that number. What is not discussed is the amount and type of validation testing that was performed to gain confidence in previous designs. Today, we are forced to design to the same, or similar, requirements but are only allowed a fraction of the validation testing. Our customers expect and demand the same or better reliability from our SRMs.

Now we are addressing how we can be more affordable. Our internal goal is to reduce the cost of an SRM by 50%. The SF and MS design approach does not readily lend itself to helping us achieve this goal. The uncertainties, which lead to increased weight and cost, are not explicitly defined but are hidden in the single point numbers. Herein we address one approach that will help identify uncertainties and reduce them, ultimately allowing us to reduce both weight and cost.

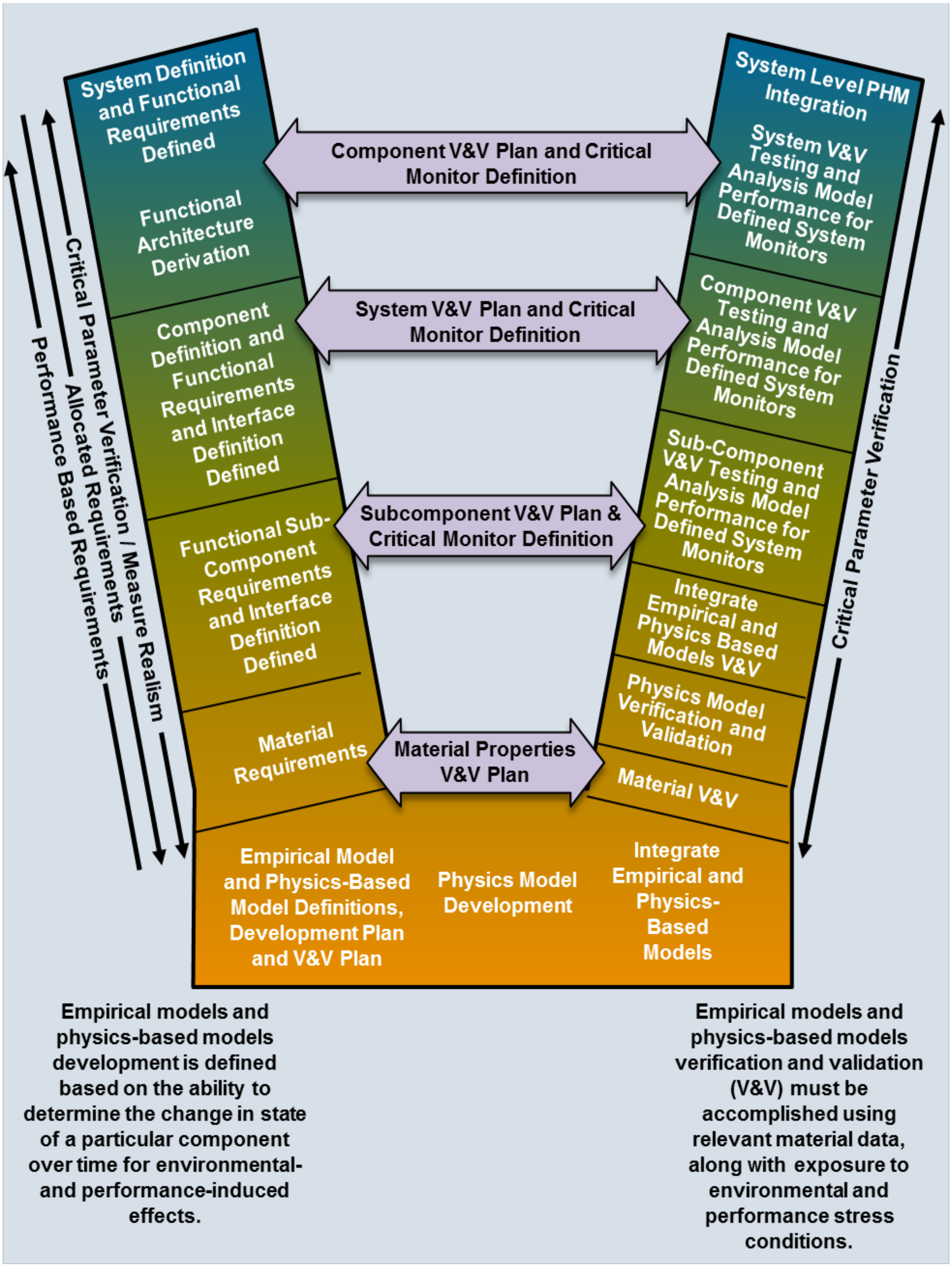

Figure 3-1. Systems Engineering V Diagram Showing Requirements Capture, Allocation, and Verification and

Validation (V&V) Process [1]

Most of the SRM development programs, i.e., Titan, Minuteman, Peacekeeper, Trident, etc., followed a design MS process that allowed them to use conservative design and analysis methods. This was probably the only option since the complexity of the materials was known but test methods and models for predicting material response and failure with confidence did not exist. For example, propellant is nonlinear and viscoelastic, meaning propellant properties vary depending on the temperature and load rate conditions they are tested under. This complexity required very extensive material characterization test matrices that subject the material to wide ranges of test temperatures and loading rates. Master curves are typically developed to allow analysts to choose material properties that are representative of the loading conditions of interest. Early designers and analysts developed "a feel" for the material response and failure behavior. The historical success of this approach is impressive and is often still followed when designing and analyzing SRMs today.

Once SRMs were produced and delivered to a customer for operational use, those customers then became very interested in tracking aging and reliability of their fleets. Figure 4-1 helps illustrate a critical difference between the historical approach for SRM design, analysis, and aging and how P_f could provide more relevant information. Figure 4-1 shows a life-cycle view whereas in reality the 0-time data is obtained during the development and production programs and the age-related data obtained by aging surveillance programs that are often not fully conceptualized until after the SRMs have been in an operational condition for some amount of time.

The top trend line in Figure 4-1 represents the material failure limit or capability. The analysis will select "a" representative capability value that is used in the numerator of the SF and MS equations whereas the full distribution of values is used in a P_f approach. Equation 4 shows a typical P_f calculation. This number is typically determined in the laboratory by testing the material or component to failure. The bottom trend line represents the loads induced onto the material as predicted through analysis. There is a slope to the induced load trend curve that may not be intuitive. An example of an induced load that can vary with time is bondline stress in an SRM.

Typically, if the propellant stiffens with age, the induced bondline stress may increase with age. This must be taken into account when addressing bondline failure modes. Induced loads are typically determined by obtaining material response properties in the laboratory and using them in a finite element model to calculate the induced loads in the SRM. This number becomes the denominator in the SF or MS equations. It is important to notice that the trend lines in Figure 4-1 pass through the Gaussian test data representations at the 50% distribution point at each age period. This figure is notional and does not accurately reflect the design MS approach. A more accurate representation would show both the capability and induced trend lines passing through the tails of the Gaussian distributions. The capability trend line would pass through the lower tail of the distribution and the induced trend line through the upper tail of its distribution. Another way to describe this is that analysts may use lower 2σ or 3σ capability properties in the SF or MS numerator and upper 2σ or 3σ induced properties in the denominator. This results in conservatism but makes quantifying uncertainty intractable since individual uncertainties are buried into a single number.

Figure 4-1 brings out other analysis subtleties that are important to recognize. The capability trend data is typically obtained by testing to failure. This testing produces time, load, and displacement data that must be converted to the failure property of interest, i.e., stress or strain. The data reduction method must be relevant to accurately predict failure but is often conservative for design and analysis purposes. The same arguments can be made for the induced load trend line (lower line in the figure). Design SF of MS calculations can be made with conservatism but P_f calculations require relevant properties and quantified uncertainties.

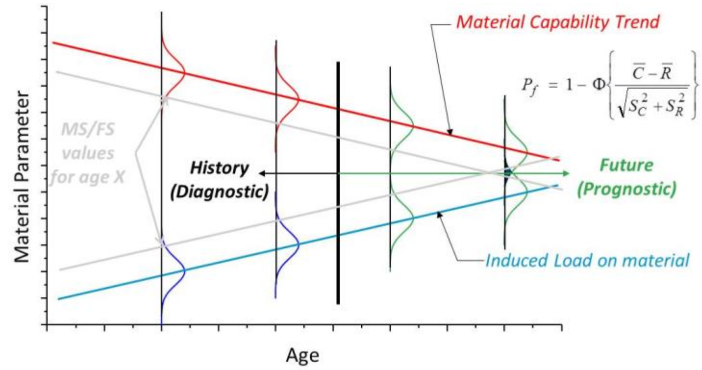

Figure 4-1. Conceptual Illustration of P_f Analysis

Figure 4-1 also illustrates another key point. The trend data must be extrapolated somehow for the approach to be predictive. There are three common ways to extrapolate data. The first is to place a best-fit curve through the existing data. This approach is not physics-based and results in low confidence in the extrapolation. Nevertheless, curve fitting is widely used in the industry because it is simple. The second approach is phenomenological extrapolation. This method entails accelerating the aging process by inducing severe environments such as elevated temperatures, temperature cycling, vibration, etc. that accelerate aging behaviors. The third method is sometimes called mechanistic since it identifies the aging mechanisms and determines the future critical parameters by evolving the aging mechanisms based on an assumed future environmental exposure. This method requires understanding of the major aging mechanisms on a micro-level and a way of relating aging mechanisms to material property evolution. All three methods are currently used in the industry to extrapolate data but the confidence associated with each method is not at all the same.

It should be obvious at this point that the use of the design SF or MS for determining SRM end of life is too conservative and does not rigorously address uncertainties. More rigorous methods that accurately address uncertainties are required to confidently address end of life decisions.

The authors believe that we, the SRM community, are currently in transition from the historical design SF and MS approach to a more rigorous approach to dealing with uncertainty. This transition is being somewhat forced by the current environment. Our traditional customers' budgets are being drastically reduced forcing Orbital ATK and other SRM manufactures to become more efficient and affordable. It can easily be argued that the conservatism of the design SF and MS approach leads to inefficient, heavy, and less affordable designs. Transitioning to a design approach that rigorously addresses P_f is where we are headed and there are a few obvious drivers.

The SRM community's adoption of systems engineering principles and approaches helps guide the transition. The previous section discussed in some detail the rigor required by the systems engineering approach where government-levied requirements are allocated down to the propulsion system and critical parameter verification is forcing us to more rigorously address uncertainty. Orbital ATK's recent self-funded project where designers and analysts were asked to quantify uncertainties in their methods is another example of what is driving the transition. Attempts to address uncertainty when using traditional design SF and MS approaches can quickly frustrate someone who does not understand the subtleties of the process.

Future papers will address more rigorous P_f calculation approaches that incorporate our knowledge of variability sources that are products of our manufacturing process as well as sources stemming from the operational life-cycle. Some examples of SRM production variability sources include cure temperatures and times. Cure temperature of 135°F ± 5°F is a common cure temperature and 72 hours ± 12 hours may represent a typical cure time. Propellant properties from a motor cured at the maximum allowable cure time and temperature will be significantly different from propellant properties from the minimum allowable. Dealing with uncertainties can be overwhelming and often lead to lumping the uncertainties into a single number. However, understanding sensitivity of the individual uncertainties enables us to more adequately address our customers' and self-imposed goals.

5. CONCLUSION

This paper provides a discussion on the importance of understanding uncertainty in our predictions. It provides a discussion on where some of the uncertainty can come from and why it is important to understand this and account for it in the predicted performance of our products. The paper provides some rationale on why this is important for development and operation of our products in the future and the need to quantify the P_f of each of the components of the SRM. This paper outlines the interaction between the systems engineering functions and the required analysis. The purpose of the paper is not to identify all areas where uncertainty creeps into our performance predictions, rather to identify a few in various areas to provide food for thought for the system engineers, component analysts, and design engineers as they design and support our products into the future. The authors would like to encourage additional quantification of uncertainty information from many authors to come forward to provide the community more insight and allow for enhanced optimization of our products keeping us competitive and enabling us to meet the future goals of our customers and management.

6. REFERENCES

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[2] Mehta, Unmeel B., Eklund, Dean R., Romero, Vicente J., Pearce, Jeffrey A., Keim, Nicholas S., "Simulation Credibility Advances in Verification, Validation, and Uncertainty Quantification", November 2016,NASA/TP-2016-219422 or JANNAF/GL-2016-0001.

[3] Smith, Ralph C., Uncertainty Quantification: Theory, Implementation, and Applications, SIAM, Philadelphia, 2014. Chapter 1 contains a good discussion of the problems with model error.